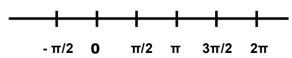
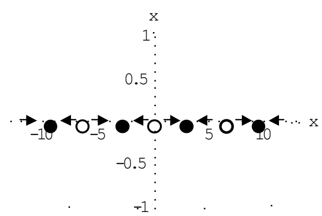
$dx/dt = \sin(x)$

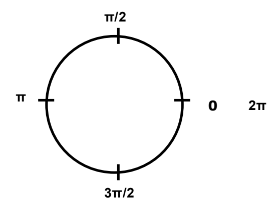
- Viewed as a flow on the line

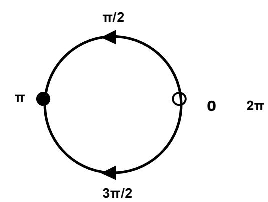


0 and 2π are not the same point



- Viewed as a Flow on the circle





$d\theta/dt = f(\theta)$

- so a 1D system oscillates only when if can be viewed as flow on a circle
- only works if $f(\theta)$ is a 2π ***periodic function***
$f(\theta)=f(\theta+2\pi)$
- This ensures that that each point on the circle has a unique velocity $d\theta/dt$
- so it doesn't work for … $d\theta/dt = \theta$

Uniform Oscillators

- Uniform Oscillator
- velocity is doesn’t vary with θ

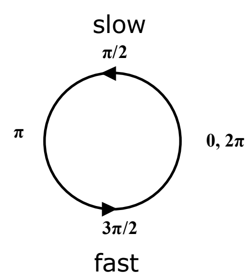
$$\dot{\theta} = \omega$$

- ω is the angular velocity
- T = 2π / ω
- T is the period

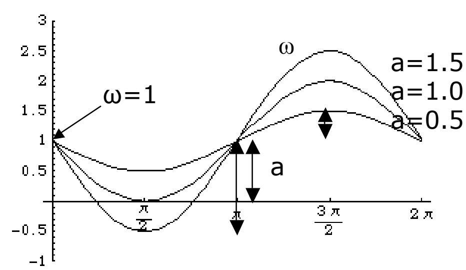
Nonuniform Oscillators

- Nonuniform Oscillator
- velocity varies with θ
- example:

$$\dot{\theta} = \omega - a\sin(\theta)$$



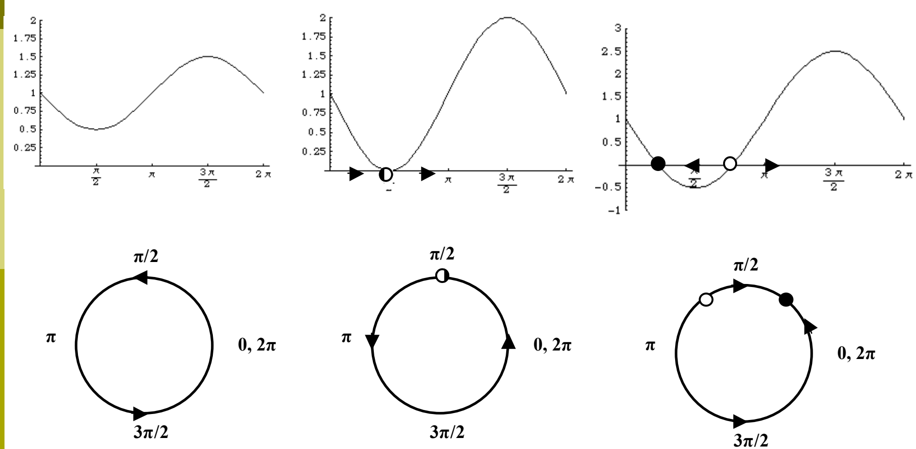
behavior depends on the value of **a** relative to **ω**



what happens at a=w?

Bifurcation at a = ω

a < ω　　　　a = ω　　　　a > ω



saddle node … and its ghost for a<w but close to a=w

Basic properties of an excitable system

1. The system contains a unique and global resting state that it returns to when not being stimulated
2. The system contains a threshold value such that when an input stimulates the system above that threshold value the phase point travels a long excursion through the phase space before returning to the rest state.

Neurons are an example of Excitable Cells

Oscillations / Rhythms Occur in Nature

- Circadian Rhythms (24 hours)
  - sleep wake cycles
- Biochemical Oscillations (1 – 20 min)
  - metabolites oscillate
- Neuronal Oscillations (ms – s)
- Cardiac Rhythms (1 s)
- Hormonal Oscillations (10 min - 24 hour)
- Communication in Animals
  - firefly